

MATHEMATICS CONTENT STANDARDS GRADE 5

By the end of fifth grade, students increase their facility with the four basic arithmetic operations applied to positive and negative numbers, fractions and decimals. They know and use common measuring units to determine length and area; they know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass in solving problems. They use grids, tables, graphs, and charts to record and analyze data.

NUMBER SENSE

Numbers determine and define quantities and relationships. They are used to make comparisons, interpret information, solve problems, and make decisions. Number sense is an understanding of number relationships. Students use estimation to make good judgments about the reasonableness of results and make sense of the many ways in which numbers are used. Number sense offers rich opportunities for investigating real-life applications and developing mathematical concepts and connections through problem solving. The number system has power that is deeper than counting, collecting and computing. Through exploration, usage and reflective thought, students construct number meaning and interpret the multiple uses encountered in the real world.

As students work with each new family of numbers (whole numbers, fractions, decimals, percents, integers, real numbers) they engage in three major tasks. First they must develop a conceptual understanding of the number family and its relationships to other sets of numbers previously studied. Then students develop meaning for the operations of addition, subtraction, multiplication and division for each family of numbers. Finally, after building a strong conceptual foundation, students must work to acquire fluency and facility with both numbers and their operations.

The acquisition of proficiency with basic facts must be encouraged and mastered, because it allows students to work confidently as they progress in their mathematical abilities. However, brain research shows that rote rehearsal may not be the best method of developing this kind of proficiency. Teachers are encouraged to use a wide variety of experiences and tasks to develop this fluency with number, while maintaining student interest and enthusiasm.

In addition, it is important for students to experience meaningful computational algorithms. Many of our standard algorithms are not consistent with the mathematical principles of place value. Students need to develop meaning during their early experiences with addition, subtraction, multiplication and division algorithms. Over time, many students will adopt the standard algorithms for convenience and efficiency. However, students using non-standard algorithms that they understand will remember and be able to use them much more effectively than students using standard algorithms that they have simply memorized. The understanding of multiplication, for example, should not be confused with fluency in using a multiplication algorithm.

Finally, while computation is a powerful part of mathematics, we need to recognize the importance of all five strands. All students must have opportunities to grow in all areas of mathematics as they continue to become proficient in computational skills.

- Students compute with very large and very small numbers, positive and negative numbers, decimals and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers.
 - Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
 - **Interpret percents as part of a hundred; find decimal and percent equivalents for common fractions; explain why they represent the same value; and compute a given percent of a whole number.**
 - Understand and compute positive integers (whole numbers and zero); powers of non-negative integers; compute examples as repeated multiplication.
 - Determine the prime factors of all numbers through 50 and write numbers as the product of their prime factors using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).
 - **Identify and represent positive and negative integers, decimals, fractions and mixed numbers on a number line.**

- Students estimate, calculate, and solve problems involving addition, subtraction and simple multiplication and division of fractions and decimals.
 - **Add, subtract, multiply and divide with decimals and negative numbers and verify the reasonableness of the results.**
 - **Demonstrate proficiency with division, including division with positive decimals and long division with multiple digit divisors.**
 - Determine the least common multiple and greatest common divisor of whole numbers. Use them to solve problems with fractions (e.g., to find a common denominator in order to add two fractions or to find the reduced form of a fraction.)
 - **Solve simple problems including ones arising in real life situations involving the addition and subtraction of fractions.**
 - Demonstrate understanding of multiplication and division of fractions by explaining the process pictorially, verbally, and in writing.
 - **Compute and perform simple multiplication and division of fractions, apply these procedures to solving problems, and express answers in simplest form.**

- **Students explore and use a variety of strategies to compute mentally.**

ALGEBRA AND FUNCTIONS

Algebra is the language of all mathematics and science, and a tool for solving problems. It is the language of operation, symbol manipulation, and variables. It describes and interprets relationships among quantities. Algebra is generalized arithmetic interwoven through all strands, and is closely connected to functions. A function is a relationship among quantities and can be represented using tables, graphs and algebraic symbols. Functions often represent a way of generalizing a numerical pattern. When there is a functional relationship between two quantities, the value of the first quantity determines the corresponding value of the second. The study of functions enables students to see relationships and to make predictions based on those relationships.

- Students describe and use variables to express a numeric, geometric, or graphical relationship, and model and solve algebraic equations with one variable.
 - Use information taken from a graph or equation to answer questions about a real life problem situation.
 - **Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.**
 - Model, describe, and use the distributive property in equations and expressions with variables.
 - Write, identify and graph ordered pairs in the four quadrants of the coordinate plane.
 - **Solve problems involving linear functions with integer values, write the equation, and graph the resulting ordered pairs of integers on a grid.**

MEASUREMENT AND GEOMETRY

Through the study of geometry, students link mathematics to space and form in the world around them and in the abstract. In this strand, the students are exposed to and investigate one-dimensional, two-dimensional and three-dimensional space by exploring shape, area, and volume; studying lines, angles, points and surfaces; and engaging in other visual and concrete experiences.

- **Students understand and compute area, perimeter, volume and surface area of simple objects.**
 - Derive and use the formula for the area of right triangles and of parallelograms by comparing with the area of rectangles (i.e. two of the same triangles make a rectangle with twice the area; a parallelogram is compared to a rectangle with the same area found by cutting and pasting a right triangle).
 - Construct cube and rectangular boxes from two-dimensional patterns and use this to compute the surface area for these objects.
 - Model and compute the volume of simple geometric solids, selecting and using appropriate units in both metric and customary systems (cubic centimeter [cm, inches], cubic meter [m³], cubic inches [in³], cubic yard [yd³]).
 - Model and solve problems by differentiating among, and using units of measure for two and three dimensional objects (perimeter, [cm, inches], area [cm², in²] and volume [cm³, in³]).
- Students identify, describe, draw and classify properties of, and relationships between, plane and solid geometric figures.
 - **Measure, identify and draw angles, perpendicular and parallel lines, rectangles and triangles, using appropriate tools (e.g., straight edge, ruler, compass, protractor or drawing software).**
 - **Determine that the sum of the angles of any triangle is 180 degrees and the sum of the angles of any quadrilateral is 360 degrees and use this information to solve problems.**
 - **Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.**

- **Students choose appropriate units (metric and U.S. customary) and tools, and estimate and measure length (to the nearest sixteenth of an inch and to the nearest mm), liquid volume and weight/mass.**

STATISTICS, DATA ANALYSIS, AND PROBABILITY

The study of statistics helps students learn to collect and organize information in a variety of graphs, charts, and tables to make the data easier for the students and others to understand. Students learn to interpret data and to make decisions based on their interpretations. Students learn probability, the study of chance, so that numerical data can be used to predict future events and outcomes.

- **Students display, analyze, compare and interpret different data sets, including data sets that are not the same size.**
 - Analyze data to determine measures of central tendency (mean, median, mode), when you might appropriately use each measure, and explain why these measures are useful in different situations.
 - Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circlegraphs) and explain which types of graphs are appropriate for different kinds of data sets.
 - Use fractions and percentages to compare data sets of different size.
 - **Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph.**

MATHEMATICAL REASONING, PROBLEM SOLVING, AND COMMUNICATION STANDARDS

The study of mathematics is much more than following procedures to determine answers to math computations and word problems. The student of mathematics is learning how to think clearly while solving problems that don't necessarily have predetermined single solutions. This skill is not only essential across all academic subject areas, but extends into virtually every career and job. More often than not, this thinking clearly must be done while working with and getting along with others, sharing information, expertise, and ideas. Frequently, reasoning has to be communicated to others, formally and informally, in writing and orally. Mathematical Reasoning is the study of thinking clearly.

Students solve problems using a 4-step process:

- **Students make decisions about how to approach problems.**
 - Identify obstacles to solving the problem; identify the largest obstacle.
 - Analyze problems by identifying relationships, discriminating relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
 - Formulate mathematical conjectures based upon a general description of the mathematical problem posed:
 - “Have I done a problem like this before?”
 - “What type of answer am I expecting?”
 - “What mathematical steps do I need to take?”
 - “How am I going to overcome the obstacles?”

- ☐ Determine when and how to break a problem into simpler parts.
- ☐ **Students use strategies, skills and concepts in finding solutions.**
 - ☐ Use estimation to predict results.
 - ☐ Choose appropriate problem-solving strategies, including but not limited to:
 - write and solve an algebraic equation
 - make a table
 - use logical reasoning
 - solve a simpler problem
 - look for a pattern
 - work backward
 - draw a diagram or graph
 - guess and check
 - make a model or simulation
 - use proportional reasoning
 - use appropriate tools and technology
 - ☐ Make precise calculations and check the validity of the results from the context of the problem.
 - ☐ Use estimation to verify the reasonableness of calculated results.
 - ☐ Make and test conjectures using both inductive and deductive reasoning.
- ☐ **Students communicate results by justifying and explaining their process and solution.**
 - ☐ Use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
 - ☐ Express the solution clearly and logically using appropriate mathematical notation, terms and clear language, and support solutions with evidence, in both verbal and symbolic work.
 - ☐ Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- ☐ **Students determine a solution is complete and move beyond a particular problem by generalizing to other situations.**
 - ☐ Evaluate the reasonableness of the solution in the context of the original situation.
 - ☐ Note method of deriving the solution and demonstrate conceptual understanding of the derivation by creating and solving similar problems.
 - ☐ Develop generalizations of the results obtained and the strategies used and extend them to new problem situations.