MATHEMATICS CONTENT STANDARDS
GRADE 4

By the end of fourth grade, students understand large numbers and addition, subtraction, multiplication and division of whole numbers. They describe and compare simple fractions and decimals. They develop their understanding of variables, formulas and linear relationships. They understand the properties of and the relationships between plane geometric figures. They collect, represent and analyze data to answer questions.

NUMBER SENSE

Numbers determine and define quantities and relationships. They are used to make comparisons, interpret information, solve problems, and make decisions. Number sense is an understanding of number relationships. Students use estimation to make good judgments about the reasonableness of results and make sense of the many ways in which numbers are used. Number sense offers rich opportunities for investigating real-life applications and developing mathematical concepts and connections through problem solving. The number system has power that is deeper than counting, collecting and computing. Through exploration, usage and reflective thought, students construct number meaning and interpret the multiple uses encountered in the real world.

As students work with each new family of numbers (whole numbers, fractions, decimals, percents, integers, real numbers) they engage in three major tasks. First they must develop a conceptual understanding of the number family and its relationships to other sets of numbers previously studied. Then students develop meaning for the operations of addition, subtraction, multiplication and division for each family of numbers. Finally, after building a strong conceptual foundation, students must work to acquire fluency and facility with both numbers and their operations.

The acquisition of proficiency with basic facts must be encouraged and mastered, because it allows students to work confidently as they progress in their mathematical abilities. However, brain research shows that rote rehearsal may not be the best method of developing this kind of proficiency. Teachers are encouraged to use a wide variety of experiences and tasks to develop this fluency with number, while maintaining student interest and enthusiasm.

In addition, it is important for students to experience meaningful computational algorithms. Many of our standard algorithms are not consistent with the mathematical principles of place value. Students need to develop meaning during their early experiences with addition, subtraction, multiplication and division algorithms. Over time, many students will adopt the standard algorithms for convenience and efficiency. However, students using non-standard algorithms that they understand will remember and be able to use them much more effectively than students using standard algorithms that they have simply memorized. The understanding of multiplication, for example, should not be confused with fluency in using a multiplication algorithm.

Finally, while computation is a powerful part of mathematics, we need to recognize the importance of all five strands. All students must have opportunities to grow in all areas of mathematics as they continue to become proficient in computational skills.

- Students understand place value of whole numbers and decimals to two places, how these relate to simple fractions, and use concepts of negative numbers.
  - **Read and write whole numbers in the millions.**
  - **Order, compare, and round whole numbers through millions and decimals to two places.**
  - **Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand or hundred thousand.**
  - Decide when a rounded solution is called for, and explain why this is the case.
  - **Interpret different meanings for fractions including:**
    - **parts of a whole**
    - **parts of a set**
    - **indicated division of whole numbers**
    - **and quantities (and measures) between whole numbers on a number line.**
  - **Write tenths and hundredths in decimal and fraction notation and know fraction/decimal equivalents for halves and fourths. (e.g., ½ = 0.5, 7/4 = 1 ¾ = 1.75)**
  - Write the fraction represented by a drawing of parts of a figure; represent a given fraction using drawings.
  - **Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, "owing").**
  - **Identify the relative position of fractions, mixed numbers, and decimals to two decimal places on a number line.**

- Students extend their use and understanding of whole numbers to addition and subtraction of simple decimals.
  - **Estimate and compute the sum or difference of whole numbers and positive decimals to two places.**
  - Round two-place decimals to the nearest tenth or the nearest whole number, and use rounding to judge the reasonableness of an answer.

- Students solve problems involving the addition, subtraction, multiplication and division of whole numbers, including the addition and subtraction of integers (positive and negative numbers).
  - Demonstrate understanding of, and the ability to use algorithms for addition and subtraction of multi-digit numbers.
  - Demonstrate understanding of, and the ability to use algorithms for multiplying a multi-digit number by a two-digit number and long division for dividing a multi-digit number by a one-digit number; use relationships between them to simplify computations and to check results.
  - **Solve problems involving multiplication of multi-digit numbers by two-digit numbers.**
  - **Solve problems involving division of multi-digit numbers by one-digit numbers.**

- **Students know how to factor small whole numbers to 100.**

- Understand that many whole numbers can be factored in different ways (e.g., 12 = 4 x 3 = 2 x 6 = 2 x 2 x 3) and that such numbers are called composite numbers.
- Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves, and that such numbers are called prime numbers.

- Students explore and use a variety of strategies to compute mentally.

ALGEBRA AND FUNCTIONS

Algebra is the language of all mathematics and science, and a tool for solving problems. It is the language of operation, symbol manipulation, and variables. It describes and interprets relationships among quantities. Algebra is generalized arithmetic interwoven through all strands, and is closely connected to functions. A function is a relationship among quantities and can be represented using tables, graphs and algebraic symbols. Functions often represent a way of generalizing a numerical pattern. When there is a functional relationship between two quantities, the value of the first quantity determines the corresponding value of the second. The study of functions enables students to see relationships and to make predictions based on those relationships.

- **Students use and interpret variables, mathematical symbols and properties to write and simplify expressions and sentences.**
  - Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate understanding and use of a concept or a variable).
  - Interpret and evaluate mathematical expressions that use parentheses.
  - Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.
  - Use and interpret formulas (e.g., Area = length times width or $A = lw$) to answer questions about quantities and their relationships.
  - Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.

- Students know how to manipulate equations.
  - Know and understand that equals added to equals are equal.
  - Know and understand that equals multiplied by equals are equal.

MEASUREMENT AND GEOMETRY

Through the study of geometry, students link mathematics to space and form in the world around them and in the abstract. In this strand, the students are exposed to and investigate one-dimensional, two-dimensional and three-dimensional space by exploring shape, area, and volume; studying lines, angles, points and surfaces; and engaging in other visual and concrete experiences.

- Students understand perimeter and area.
  - **Measure the area of rectangular shapes, using appropriate units: square centimeter, square meter, square kilometer, square inches, square yard, and square mile.**
  - Recognize that rectangles having the same area can have different perimeters.

- Understand that the same number can be the perimeter of different rectangles, each having a different area.
- **Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use these formulas to find the area of more complex figures by dividing them into parts with these basic shapes.**

- Students use two-dimensional coordinate grids to represent points and graph lines and simple figures.
  - **Draw the points corresponding to linear relationships on graph paper (e.g., draw the first ten points for the equation *y = 3x* and connect them using a straight line).**
  - Understand that the length of a horizontal line segment equals the difference of the *x*-coordinates
  - Understand that the length of a vertical line segment equals the difference of the *y*-coordinates

- **Students demonstrate understanding of plane and solid geometric objects. They use this knowledge to show relationships and solve problems.**
  - Identify lines that are parallel and perpendicular.
  - Identify the radius and diameter of a circle.
  - Identify congruent and similar figures.
  - MG3.4 Identify figures that have bilateral and rotational symmetry.
  - Know the definitions of right, acute, and obtuse angles. Understand that 90, 180, 270, and 360 degrees are, respectively, associated with ¼, ½, ¾ and full turns.
  - **Visualize, describe and represent geometric solids (e.g. prisms, pyramids, etc.) in terms of the number and shape of faces, edges and vertices; interpret two-dimensional representations of three-dimensional objects; draw patterns (of faces) for a solid that when folded will make a model of the solid.**
  - Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their features.
  - Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, and trapezoid).

STATISTICS, DATA ANALYSIS, AND PROBABILITY

The study of statistics helps students learn to collect and organize information in a variety of graphs, charts, and tables to make the data easier for the students and others to understand. Students learn to interpret data and to make decisions based on their interpretations. Students learn probability, the study of chance, so that numerical data can be used to predict future events and outcomes.

- **Students organize, represent and interpret numerical data, and clearly communicate their findings.**
  - Formulate survey questions, systematically collect and represent data on a number line, and coordinate graphs, tables and charts.

- Identify the mode for sets of categorical data, and the mean, mode, median, range, and any apparent outliers for numerical data sets.
- Interpret one- and two-variable data graphs to answer questions about a situation.

- Students make predictions for simple probability situations in an organized way.
  - **Represent all possible outcomes for a simple probability situation in an organized way (e.g., tree diagrams, line plots, grids, tables).**
  - Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; ¾).

MATHEMATICAL REASONING, PROBLEM SOLVING, AND COMMUNICATION STANDARDS

The study of mathematics is much more than following procedures to determine answers to math computations and word problems. The student of mathematics is learning how to think clearly while solving problems that don't necessarily have predetermined single solutions. This skill is not only essential across all academic subject areas, but extends into virtually every career and job. More often than not, this thinking clearly must be done while working with and getting along with others, sharing information, expertise, and ideas. Frequently, reasoning has to be communicated to others, formally and informally, in writing and orally. Mathematical Reasoning is the study of thinking clearly.

Students solve problems using a 4-step process:

- **Students make decisions about how to approach problems.**
  - Identify obstacles to solving the problem; identify the largest obstacle.
  - Analyze problems by identifying relationships, discriminating relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
  - Formulate mathematical conjectures based upon a general description of the mathematical problem posed:
    - "Have I done a problem like this before?"
    - "What type of answer am I expecting?"
    - "What mathematical steps do I need to take"
    - "How am I going to overcome the obstacles?"
  - Determine when and how to break a problem into simpler parts.

MR2.0: Students use strategies, skills and concepts in finding solutions.

- Use estimation to predict results.
- Choose appropriate problem-solving strategies, including but not limited to:
  - write and solve an algebraic equation
  - make a table
  - use logical reasoning
  - solve a simpler problem
  - look for a pattern

- work backward
- draw a diagram or graph
- guess and check
- make a model or simulation
- use proportional reasoning
- use appropriate tools and technology

- Make precise calculations and check the validity of the results from the context of the problem.
- Use estimation to verify the reasonableness of calculated results.
- Make and test conjectures using both inductive and deductive reasoning.

- **Students communicate results by justifying and explaining their process and solution.**
  - Use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
  - Express the solution clearly and logically using appropriate mathematical notation, terms and clear language, and support solutions with evidence, in both verbal and symbolic work.
  - Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

- **Students determine a solution is complete and move beyond a particular problem by generalizing to other situations.**
  - Evaluate the reasonableness of the solution in the context of the original situation.
  - Note method of deriving the solution and demonstrate conceptual understanding of the derivation by creating and solving similar problems.
  - Develop generalizations of the results obtained and the strategies used and extend them to new problem situations.