

MATHEMATICS CONTENT STANDARDS GRADE 6

By the end of sixth grade, students have mastered the four arithmetic operations with positive and negative numbers, whole numbers, fractions and decimals; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concept of and how to calculate the range, mean, median and mode of data sets. They analyze data and sampling processes for possible bias and misleading conclusions, and they use addition and multiplication of fractions routinely to calculate probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in representing an unknown part of a ratio. They solve 1-step linear equations.

NUMBER SENSE

Numbers determine and define quantities and relationships. They are used to make comparisons, interpret information, solve problems, and make decisions. Number sense is an understanding of number relationships. Students use estimation to make good judgments about the reasonableness of results and make sense of the many ways in which numbers are used. Number sense offers rich opportunities for investigating real-life applications and developing mathematical concepts and connections through problem solving. The number system has power that is deeper than counting, collecting and computing. Through exploration, usage and reflective thought, students construct number meaning and interpret the multiple uses encountered in the real world.

As students work with each new family of numbers (whole numbers, fractions, decimals, percents, integers, real numbers) they engage in three major tasks. First they must develop a conceptual understanding of the number family and its relationships to other sets of numbers previously studied. Then students develop meaning for the operations of addition, subtraction, multiplication and division for each family of numbers. Finally, after building a strong conceptual foundation, students must work to acquire fluency and facility with both numbers and their operations.

The acquisition of proficiency with basic facts must be encouraged and mastered, because it allows students to work confidently as they progress in their mathematical abilities. However, brain research shows that rote rehearsal may not be the best method of developing this kind of proficiency. Teachers are encouraged to use a wide variety of experiences and tasks to develop this fluency with number, while maintaining student interest and enthusiasm.

In addition, it is important for students to experience meaningful computational algorithms. Many of our standard algorithms are not consistent with the mathematical principles of place value. Students need to develop meaning during their early experiences with addition, subtraction, multiplication and division algorithms. Over time, many students will adopt the standard algorithms for convenience and efficiency. However, students using non-standard algorithms that they understand will remember and be able to use them much more effectively than students using standard algorithms that they have simply memorized. The understanding of multiplication, for example, should not be confused with fluency in using a multiplication algorithm.

Finally, while computation is a powerful part of mathematics, we need to recognize the importance of all five strands. All students must have opportunities to grow in all areas of mathematics as they continue to become proficient in computational skills.

- Students compare and order fractions, decimals, and mixed numbers. They solve problems involving fractions, ratios, proportions, and percentages.
 - **Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.**
 - **Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities using appropriate notations (a/b , a to b , $a:b$).**
 - **Use proportional reasoning to solve problems (e.g. determine the value of N if $4/7 = N/21$).**
 - **Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned and tips.**

- Students estimate, calculate and solve problems involving addition, subtraction, multiplication and division of rational numbers, positive and negative integers, and positive fractions and decimals.
 - **Solve problems involving addition, subtraction, multiplication and division of fractions and explain why a particular operation was used for a given situation.**
 - Explain the meaning of multiplication and division of fractions and perform the calculations including canceling.
 - **Model and solve addition, subtraction, multiplication and division problems in contexts that use positive and negative numbers.**
 - Determine the least common multiple and greatest common divisor of whole numbers. Use them to solve problems.

- **Students use a variety of strategies to calculate mentally in order to develop a keen number sense and increase facility and fluency of computation.**
 - Students divide multiples of ten by canceling zeroes.
 - Students use compatible numbers to simplify multi-step multiplication.
 - Students will use compensation strategies when appropriate.

ALGEBRA AND FUNCTIONS

Algebra is the language of all mathematics and science, and a tool for solving problems. It is the language of operation, symbol manipulation, and variables. It describes and interprets relationships among quantities. Algebra is generalized arithmetic interwoven through all strands, and is closely connected to functions. A function is a relationship among quantities and can be represented using tables, graphs and algebraic symbols. Functions often represent a way of generalizing a numerical pattern. When there is a functional relationship between two quantities, the value of the first quantity determines the corresponding value of the second. The study of functions enables students to see relationships and to make predictions based on those relationships.

- **Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations and graph and interpret their results.**
 - **Write and show all steps involved in solving one-step linear equations in one variable.**
 - Write and evaluate an algebraic expression for a given situation using formulas with up to three variables (e.g., $A = lw$).
 - Apply algebraic order of operations and the commutative, associative and distributive properties to evaluate expressions and explain each step in the process.
 - Solve problems using correct order of operations manually and by using a scientific calculator.

- Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions.
 - **Convert one unit of measurement to another (e.g. from feet to miles, from centimeters to inches).**
 - Demonstrate understanding that rate is a measure of one quantity per unit value of another quantity.
 - **Solve problems involving rates, average speed, distance and time.**

- **Students investigate geometric patterns and describe them algebraically.**
 - Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2}bh$, $C = \pi d$) which give the perimeter of a rectangle, area of a triangle, and circumference of a circle respectively.
 - Express simple relationships arising from geometry in symbolic form.

MEASUREMENT AND GEOMETRY

Through the study of geometry, students link mathematics to space and form in the world around them and in the abstract. In this strand, the students are exposed to and investigate one-dimensional, two-dimensional and three-dimensional space by exploring shape, area, and volume; by studying lines, angles, points and surfaces; and by engaging in other visual and concrete experiences.

- Students deepen their understanding of measurement of plane and solid shapes and use this understanding to solve problems.
 - **Understand the concept of a constant number like pi. Know and use the formula for the circumference and area of a circle.**
 - Determine estimates of pi (3.14; $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.
 - Know how to use the formulas for the volume of triangular prisms and cylinders (area of base x height); compare and explain the connections between these formulas and the formula for the volume of a rectangular solid.

- Students identify and describe the properties of two-dimensional figures.
 - Identify angles as vertical, adjacent, complementary, and/or supplementary, and provide descriptions of these terms.
 - **Use the properties of complementary and supplementary angles and of the angles of a triangle to solve problems involving an unknown angle.**
 - Draw quadrilaterals and triangles given information about them (e.g. a quadrilateral having equal sides but no right angles, a right isosceles triangle).
 - Students explore mathematical phenomena and make hypotheses about the geometric relationships that exist
 - Explore the concepts of reflections and line symmetry.
 - Use reflections and rotations to make tessellations of geometric and Escher-like figures.
 - Interpret and draw two-dimensional representations of three-dimensional objects.

STATISTICS, DATA ANALYSIS, AND PROBABILITY

The study of statistics helps students learn to collect and organize information in a variety of graphs, charts, and tables to make the data easier for the students and others to understand. Students learn to interpret data and to make decisions based on their interpretations. Students learn probability, the study of chance, so that numerical data can be used to predict future events and outcomes.

- **Students compute and analyze statistical measurement for data sets.**
 - Compute the range, median, mean and mode of data sets.
 - Understand how additional data added to data sets can affect these computations of measures of central tendency.
 - Understand how the inclusion or exclusion of outliers affect measures of central tendency.
 - **Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.**
- **Students use data samples of a population and describe the characteristics and limitations of the samples.**
 - Compare different samples from a population with the data from the entire population and identify when it makes sense to use a sample.
 - Identify different ways of selecting a sample (e.g., convenience sampling, those who respond to a survey, random sampling) and which makes a sample more representative for a population.
 - Analyze data displays and explain how the way the question was asked might have influenced the results obtained, and/or how the way results were displayed might have influenced the conclusions reached.
 - Identify data that represent sampling and explain why the sample (and the display) may be biased.
 - Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

- **Students determine theoretical and experimental probabilities and use these to make predictions about events.**
 - Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.
 - Use data to estimate the probability for future events (e.g., batting averages or number of accidents per mile driven).
 - Represent probabilities as ratios, proportions, decimals between 0 and 1, percents between 0 and 100, and check that probabilities computed are reasonable; know how this is related to the probability of an event not occurring.
 - Investigate and compare situations where the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.
 - **Understand the difference between independent and dependent events and how this affects the results for specific probability situations.**

MATHEMATICAL REASONING, PROBLEM SOLVING, AND COMMUNICATION STANDARDS

The study of mathematics is much more than following procedures to determine answers to math computations and word problems. The student of mathematics is learning how to think clearly while solving problems that don't necessarily have predetermined single solutions. This skill is not only essential across all academic subject areas, but extends into virtually every career and job. More often than not, this thinking clearly must be done while working with and getting along with others, sharing information, expertise, and ideas. Frequently, reasoning has to be communicated to others, formally and informally, in writing and orally. Mathematical Reasoning is the study of thinking clearly.

Students solve problems using a 4-step process:

- **Students make decisions about how to approach problems.**
 - Identify obstacles to solving the problem; identify the largest obstacle.
 - Analyze problems by identifying relationships, discriminating relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
 - Formulate mathematical conjectures based upon a general description of the mathematical problem posed:
 - “Have I done a problem like this before?”
 - “What type of answer am I expecting?”
 - “What mathematical steps do I need to take”
 - “How am I going to overcome the obstacles?”
 - Determine when and how to break a problem into simpler parts.
- **Students use strategies, skills and concepts in finding solutions.**
 - Use estimation to predict results.
 - Choose appropriate problem-solving strategies, including but not limited to:
 - write and solve an algebraic equation

- make a table
 - use logical reasoning
 - solve a simpler problem
 - look for a pattern
 - work backward
 - draw a diagram or graph
 - guess and check
 - make a model or simulation
 - use proportional reasoning
 - use appropriate tools and technology
- ⇒ Make precise calculations and check the validity of the results from the context of the problem.
 - ⇒ Use estimation to verify the reasonableness of calculated results.
 - ⇒ Make and test conjectures using both inductive and deductive reasoning.
- **Students communicate results by justifying and explaining their process and solution.**
 - ⇒ Use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
 - ⇒ Express the solution clearly and logically using appropriate mathematical notation, terms and clear language, and support solutions with evidence, in both verbal and symbolic work.
 - ⇒ Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- **Students determine a solution is complete and move beyond a particular problem by generalizing to other situations.**
 - ⇒ Evaluate the reasonableness of the solution in the context of the original situation.
 - ⇒ Note method of deriving the solution and demonstrate conceptual understanding of the derivation by creating and solving similar problems.
 - ⇒ Develop generalizations of the results obtained and the strategies used and extend them to new problem situations.